Mathematical Challenges in Quantum Mechanics

Third School and Workshop

June 13 – 18, 2022, Como (Italy)

Program & Abstracts

Schedule

	Sala Bianca, Via Vincenzo Bellini 1			DiSAT, Via Valleggio 11		
	Lunedi	Martedi	Mercoledi	Giovedi	Venerdi	Sabato
08:30	Opening					
09:00	Laptev	Weinstein	Séré	Séré	Laptev	Laptev
10:00	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
10:30	Weinstein	Séré	Weinstein	Weinstein	Laptev	Laptev
11:30	Séré	Séré	Weinstein	Weinstein	Lunch Break	Behrndt
12:30	Lunch Break	Lunch Break	Lunch Break	Lunch Break		Closing
14:30	Séré	Jex	Laptev	Cao	Basti	
14:50		Hearnshaw		Gallone	Ferretti	
15:10		Black		Cassano	Saberbaghi	
15:30	Kouande	Malinovitch	Lauritsen	Holzmann	Fermi	
15:50	Wessel	Recovery Pause	Roos	Poster Session	Recovery Pause	
16:10	Henheik	Cazalis	Nguyen		Borrelli	
16:30	Recovery Pause	Bezerra de Matos	Recovery Pause		Boni	
16:50	Carlen	Merz	Schraven	Coffee Break	Scandone	
17:10		Rout	Siebert		Kosche	
17:30	ends at 17:50	Coffee Break	Coffee Break	Cenatiempo	Coffee Break	
18:00	Welcome Cocktail	Porta	Nonnenmacher		Mazzucchi	
18:30						
19:00						
19:30						

Courses

Ari Laptev – Spectral Theory and its Applications
Eric Séré – A Rigorous Approach to some Nonlinear Models from Relativistic Quantum Mechanics
Michael I. Weinstein – Continuum Models of Graphene and Analogous Materials

Plenary Talks

Jussi Behrndt – The generalized Birman-Schwinger principle
Eric Carlen – The rate of approach to equilibrium for some many-body Lindblad equations
Serena Cenatiempo – The excitation spectrum of two dimensional Bose gases in the Gross-Pitaevskii regime
Sonia Mazzucchi – Mathematical challenges in Feynman path integrals
Stéphane Nonnenmacher – Quantum resonances in presence of classical chaos
Marcello Porta – Effective dynamics of extended Fermi gases in the high-density regime

Contributed Talks

Giulia Basti – Regularized zero-range Hamiltonian for three bosons: construction and approximation
Rodrigo Bezerra de Matos – Irreducibility of the Bloch Variety for Finite-Range Schrödinger Operators
Adam Black – Scattering for Schrödinger operators with potentials concentrated near a subspace
Filippo Boni – Ground states of the 2D nonlinear Schrödinger equation with a delta interaction
William Borrelli – Complete ionization for a non autonomous two-dimensional point interaction
Jinghao Cao – Non-reciprocal wave propagation in space-time modulated media
Biagio Cassano – General δ-shell interactions for the two-dimensional Dirac operator
Jean Cazalis – Dirac cones for nonlinear periodic Schrödinger operators at dissociation
Davide Fermi – The semiclassical limit with zero-range potentials in one dimension
Daniele Ferretti – Regularized zero-range Hamiltonian for a Bose gas with an impurity
Matteo Gallone – Self-adjoint Laplacians on the Grushin cylinder
Peter Hearnshaw – Analyticity of the one-particle density matrix for Coulombic wavefunctions
Joscha Henheik – Local perturbations perturb locally in weakly interacting quantum spin systems.
Markus Holzmann – Spectral transitions for Dirac operators with δ-shell potentials supported on curves
Michal Jex – Quantum systems at the brink: existence and decay rates of bound states at thresholds
Thea Kosche – Asymptotic Floquet theory and its application to perturbation control of Floquet exponents and classification of asymptotic exceptional points
Adechola Emile Kodjo Kouande – Phase Transition in the Peierls model for polyacetylene
Asbjørn Bækgaard Lauritsen – The BCS energy gap at low and high density
Tal Malinovitch – Scattering for Schrödinger operators with potentials concentrated near a subspace
Konstantin Merz – Random Schrödinger operators with complex decaying potentials
Dinh Thi Nguyen – Thomas–Fermi profile of a fast rotating Bose–Einstein condensate
Barbara Roos – Boundary Superconductivity in the BCS Model
Andrew James Rout – A microscopic derivation of Gibbs measures for the 1D focusing cubic nonlinear Schrödinger equation
Hamidreza Saberbaghi – Point interaction in quantum mechanics revised
Raffaele Scandone – Spectral properties for three-dimensional Schrödinger operators with point interactions
Severin Stefan Schraven – Bogoliubov Theory for Trapped Bosons in the Gross-Pitaevskii Regime
Oliver Siebert – Bose-Einstein condensation on hyperbolic spaces
Tom Wessel – Lieb-Robinson bounds and automorphic equivalence for long-range interactions

Poster Session

Giuliano Angelone – Hearing the shape of a quantum boundary condition
Olena Atlasiuk – On linear inhomogeneous boundary-value problems for differential systems in sobolev spaces
Yonah Jacob Borns-Weil – The spectrum of an almost maximally open quantized cat map
Florian Michael Haberberger – N-particle Bose Gas on a box with Neumann boundary conditions
Viktoriia Krechko – On solutions of evolution equations describing the propagation of correlations in quantum systems in a mean field approximation
Henry Martin – Thermodynamic stable site for interstitial solute (N or O) in bcc-refractory metals (Mo and Nb) using density functional theory
Izak Mendel Oltman – Probabilistic Weyl law for perturbed Berezin-Toeplitz operators
Henry Elorm Quarshie – The atomic and electronic structures of Si-Ge interfaces: first princples calculations
Christian Stelzer – Dirac operators with singular interactions on Lipschitz hypersurfaces
Georg Stenzel – Schrödinger operators with oblique transmission conditions in $\mathbb{R}^2$

Abstracts

Plenary Talks

The generalized Birman-Schwinger principle

Jussi Behrndt
Technische Universität Graz

In this lecture we discuss a generalized Birman–Schwinger principle in the non-self-adjoint context. In particular, we provide a detailed discussion of geometric and algebraic multiplicities of eigenvalues of the basic operator of interest (e.g., a Schrödinger operator) and the associated Birman–Schwinger operator, and additionally offer a careful study of the associated Jordan chains of generalized eigenvectors of both operators. This talk is based on a joint paper with Fritz Gesztesy and Tom ter Elst.

The rate of approach to equilibrium for some many-body Lindblad equations

Eric Carlen
Rutgers University

We study a class of Lindblad equations describing binary collisions of a many body quantum system and present results on spectral gaps and entropy dissipation.
This is joint work with Michael Loss.

The excitation spectrum of two dimensional Bose gases in the Gross-Pitaevskii regime

Serena Cenatiempo
GSSI

We consider a system of N bosons in the two-dimensional unit torus, modelling a trapped Bose gas where both the quantum and thermal motions are frozen in one direction. We assume particles to interact through a repulsive two-body potential, with a scattering length that is exponentially small in N (Gross-Pitaevskii regime). In this scaling limit - which provides a first example of a dilute regime in which peculiarities of two-dimensional systems can be observed - we establish the validity of the predictions of Bogoliubov theory, determining the ground state energy of the Hamilton operator and its low-energy excitation spectrum, up to errors that vanish in the limit $N \to \infty$.
Based on joint works with Cristina Caraci and Benjamin Schlein.

Mathematical challenges in Feynman path integrals

Sonia Mazzucchi
Universitá di Trento

Since their introduction in the early 40s, Feynman path integrals have always been a powerful tool for theoretical physics on the one hand and a mathematical challenge on the other. Despite decades of effort, a definitive mathematical theory of Feynman path integration is still missing and, while some steps have been taken in this direction, there are fundamental issues that still deserve further investigation. In this talk I shall give an overview of this topic with a historical perspective, highlighting recent developments and some open problems.

Quantum resonances in presence of classical chaos

Stéphane Nonnenmacher
Université Paris-Sud

We consider scattering of scalar waves by hard obstacles or potentials, in the high frequency (or semiclassical) regime. Beside a continuous spectrum. the quantum Hamiltonian also admits a discrete spectrum of resonances (generalized eigenvalues), which influence the long time evolution. We aim to quantitatively describe this resonance spectrum, taking into account the underlying classical dynamics (for instance, the billiard flow outside of the obstacles).
We focus on situations where this classical dynamics is chaotic: the trapped trajectories are all unstable, and form a fractal set in phase space. We will show how both the instability and the "complexity" of this trapped set influence the semiclassical distribution of the resonances.
The talk will be a survey of results obtained jointly with M. Zworski, J. Sjöstrand, and more recently by L. Vacossin.

Effective dynamics of extended Fermi gases in the high-density regime

Marcello Porta
University Paris-Sud

I will discuss the evolution of many-body Fermi gases in three dimensions, in large domains. In many physically relevant applications, due to the large number of particles involved, it is essentially impossible to extract quantitative information on the dynamics from the solution of the Schrödinger equation. However, in suitable scaling regimes one expects that the dynamics of local observables can be effectively studied via nonlinear evolution equations, which involve much less degrees of freedom, and where the effect of all particles on a single one is taken into account by a self-consistent potential. A famous example of such non-linear dynamics is the Hartree-Fock equation, for the evolution of many-body Fermi gases. In the last years, there has been important progress in the derivation of the Hartree-Fock equation from the Schrödinger equation, in the mean-field regime. The main limitation is that this regime does not allow to study extended systems: the initial datum is prepared in a volume of order 1, which means that the density is of the order of the total number of particles, and the interaction is scaled with the inverse of the number of particles. In this talk I will discuss the dynamics of many-body Fermi gases, in a joint semiclassical/high density regime, in arbitrarily large domains. This setting can be viewed as a Kac limit for interacting fermions. For a class of initial data describing zero-temperature states I will prove that, as the density goes to infinity, the many-body evolution of the reduced one-particle density matrix converges to the solution of the time-dependent Hartree equation, with a rate of convergence that is independent of the volume of the system. The result holds provided a suitable non-concentration estimate for the density of the system holds true, which we establish for short macroscopic times. The result is the natural extension of previous work on the mean-field regime to the setting of Fermi gases in the thermodynamic limit. Joint work with Luca Fresta (University of Bonn) and Benjamin Schlein (University of Zürich).

Contributed Talks

Regularized zero-range Hamiltonian for three bosons: construction and approximation

Giulia Basti
GSSI

We consider a system of three identical bosons interacting through zero- range interactions appropriately regularized by an effective three-body force to avoid the fall to the center phenomenon, known as Thomas effect, emerging in the standard TMS Hamiltonian. We prove that the Hamiltonian describing the system is self-adjoint and bounded from below. Furthermore, we show that it can be obtained as the norm resolvent limit of approximating Hamiltonians with rescaled non local interactions (separable potentials) and with a suitably renormalized coupling constant.
Based on a joint work with C. Cacciapuoti, D. Finco and A. Teta.

Irreducibility of the Bloch Variety for Finite-Range Schrödinger Operators

Rodrigo Bezerra de Matos
Texas A&M University

I will present results based on joint work with Jake Fillman and Wencai Liu arXiv:2107.06447. There we study the dispersion relation, also known as the Bloch variety, of discrete Schrödinger operators associated with a complex periodic potential and a finite-range interaction. Our main result shows that the Bloch variety is irreducible, as an analytic set, for a wide class of lattice geometries in arbitrary dimension. Examples include the square and triangular lattices as well as the extended Harper lattice. In particular, this shows that the discrete version of a conjecture of Kuchment holds for the above lattices.

Scattering for Schrödinger operators with potentials concentrated near a subspace

Adam Black
Yale University

We consider the scattering properties of Schrödinger operators with potentials concentrated near a subspace of $\mathbb{R}^d$. This is one of many models of a quantum particle interacting with a surface. For such operators, we show the existence of scattering states and characterize their orthogonal complement as a set of "surface states
, which consists of states that are confined to the subspace (such as pure point states) and states that escape it at a sublinear rate, in a suitable sense. Our proof uses a novel

interpretation of the Enss method [1] in order to obtain a dynamical characterisation of the orthogonal complement of the scattering states. In this talk we will state our results, and sketch some of the main ideas in the proof.
This is a joint work with Tal Malinovitch.

[1] Volker Enss. Asymptotic completeness for quantum mechanical potential scattering. Communications in Mathematical Physics, 61(3):285–291, 1978.

Ground states of the 2D nonlinear Schrödinger equation with a delta interaction

Filippo Boni
Università di Napoli Federico II

We investigate the existence of ground states at fixed mass of the subcritical nonlinear Schrödinger equation with a delta interaction in dimension two. We prove that ground states exist for every positive mass and show a logarithmic singularity at the interaction point. Moreover, up to a multiplication by a phase factor, they are positive, radially symmetric, and decreasing along the radial direction. In order to overcome the obstacles arising from the uncommon structure of the energy space, we prove an ad hoc result on rearrangements and move to the study of the minimizers of the action functional on the Nehari manifold, relying on its connection with the original problem.
This is a joint work with R. Adami, R. Carlone and L. Tentarelli.

Complete ionization for a non autonomous two-dimensional point interaction

William Borrelli
Università Cattolica del Sacro Cuore

In this talk I will consider the two dimensional Schrödinger equation, namely

$$i\frac{\partial\psi}{\partial t} = H(t)\psi, \tag{1}$$

where, at any fixed time t, the operator $H(t)$ is the point interaction formally given by

$$\text{“}H(t) := -\Delta + \alpha(t)\delta(\mathbf{x})\text{”}, \qquad \alpha : \mathbb{R} \to \mathbb{C}.$$

Our main purpose is to establish *complete ionization* for the evolution of (1). Global well-posedness of the Cauchy problem associated with (1) can be proved, under general assumptions on α and on the initial datum. Then, in the *monochromatic* case

$$\alpha(t) = \alpha_0 \sin(\omega t + \eta) + c, \qquad \alpha_0 \in \mathbb{R}\backslash\{0\}, \omega > 0, \eta, c \in \mathbb{R},$$

we investigate the asymptotic behavior of the survival probability of the L^2-normalized bound state associated with the sole eigenvalue λ of $H(0)$, denoted by φ. That is, we study the behavior of $|\langle\varphi,\psi(t)\rangle_{L^2}|^2$, as $t\to+\infty$, where $\psi(t,\cdot)$ is the solution of (1) with $\psi(0,\cdot)=\varphi$. Assuming the no-resonance condition

$$-\frac{\lambda}{\omega}\notin\mathbb{N},$$

we show that the survival probability vanishes, as $t\to+\infty$, and establish its decay rate up to lower order terms, thus describing the claimed ionization phenomenon.
Joint work with Raffaele Carlone (Federico II, Napoli) and Lorenzo Tentarelli (Politecnico di Torino).

[1] Borrelli W., Carlone R., Tentarelli L., Complete ionization for a non-autonomous point interaction model in d = 2, submitted, preprint: arXiv:2108.06564 (2021).

Non-reciprocal wave propagation in space-time modulated media

Jinghao Cao
ETH Zurich

We prove the possibility of achieving non-reciprocal wave propagation in space-time modulated media and give an asymptotic analysis of the non-reciprocity property in terms of the amplitude of the time-modulation. Such modulation causes a folding of the band structure of the material, which may induce degenerate points. By breaking time-reversal symmetry, we show that these degeneracies may open into non-symmetric, unidirectional band gaps. Finally, we illustrate our results by several numerical simulations.

Link to the related paper: https://arxiv.org/abs/2109.07220 (to appear on SIAM MMS)

General δ-shell interactions for the two-dimensional Dirac operator

Biagio Cassano
UniCampania

In this talk we will consider the two-dimensional Dirac operator with general local singular interactions supported on a closed curve. A systematic study of the interaction is performed by decomposing it into a linear combination of four elementary interactions: electrostatic, Lorentz scalar, magnetic, and a fourth one which can be absorbed by using unitary transformations. We address the self-adjointness and the spectral description of the underlying Dirac operator, and moreover we describe its approximation by Dirac operators with regular potentials.
This is a joint work with V. Lotoreichik, A. Mas and M. Tušek.

Dirac cones for nonlinear periodic Schrödinger operators at dissociation

Jean Cazalis
CNRS - Université Paris-Dauphine

We consider a Schrödinger operator $H = -\Delta + V_L$ acting on $L^2(\mathbb{R}$ with a potential V_L which is assumed to be periodic with respect to some lattice $\mathscr{L}_L = L\mathscr{L}$ with length scale L. Under some assumptions which covers periodic reduced Hartree-Fock theory, we show that, in the limit $L \to \infty$, the low-lying spectral bands of H_L are given to leading order by the tight-binding model. In addition, when the underlying lattice is the honeycomb lattice and under a non-degeneracy condition, we show that the dispersion relation also presents conical singularities (Dirac points) at the vertices of the Brillouin zone.

The semiclassical limit with zero-range potentials in one dimension

Davide Fermi
Università Roma Tre

We discuss the semiclassical limit for quantum particles on the line in presence of delta and delta-prime zero-range potentials. Inspired by previous works of Hagedorn dealing with regular potentials, we consider rescaled coherent states which have sharply localized positions and momenta in the classical limit. Our strategy relies on comparing the quantum dynamics with the semiclassical ones generated by singular perturbations of the free classical Liouville operator, which we construct using resolvent techniques. We derive explicit error estimates for the unitary evolutions of the said coherent states, as well as for the related wave operators and scattering matrices. Analogous results for a star graph with Kirchhoff conditions at the vertex are also mentioned.
Based on joint work with Claudio Cacciapuoti and Andrea Posilicano (University of Insubria).

Regularized zero-range Hamiltonian for a Bose gas with an impurity

Daniele Ferretti
Sapienza Univesità di Roma

We discuss a three-dimensional gas of N bosons interacting via a regularized contact interaction only with an impurity of different mass. This regularization is built adopting a proper three-body repulsion. The so-called Minlos-Faddeev extension of the free Hamiltonian that encodes such a regularization has proven to be self- adjoint and bounded from below. The typical Thomas collapse that occurs in these three-dimensional bosonic systems is therefore healed with the use of a three-body interaction.

Self-adjoint Laplacians on the Grushin cylinder

Matteo Gallone
SISSA

When a quantum particle is constrained on an orientable Riemannian manifold, one challenging problem that arises naturally is the question of the so-called 'geometric quantum confinement'. This is the possibility that a particle whose initial wavefunction is supported inside some portion of space may remain confined in such a region for all times when evolving according to the unitary group generated by the free Hamiltonian. This occurrence is related to the presence of singularities in the metric and to the (essential) self-adjointness of the Laplace-Beltrami. The prototypical example of space exhibiting this phenomenon is the 'Grushin-like cylinder' that is, roghly speaking, a cylinder with metric $ds^2 = dx^2 + |x|^{-2\alpha} dy^2$. In this talk I will consider this manifold and I will present the classification of a physically interesting sub-family of self-adjoint realisations of the Laplace-Beltrami operator in the regime where it is not essentially self-adjoint. I will discuss the advantages of the usage of Krein-Vishik-Birman self-adjoint extension theory.
This is based on a series of joint works with A. Michelangeli and E. Pozzoli.

Analyticity of the one-particle density matrix for Coulombic wavefunctions

Peter Hearnshaw
University College London

By averaging over positions of all but one of the electrons, both the electron density and the density matrix for Coulombic wavefunctions are convenient objects to understand how electrons are distributed around an atom or molecule. S. Fournais, M+T Hoffmann-Ostenhof and T.Ø. Sørensen previously proved that the electron density was real analytic away from the nuclei. We extend their methods to show real analyticity of the density matrix away from the nuclei and the diagonal. The lack of analyticity on the diagonal is interesting, and we study this further with pointwise bounds to the derivatives in the vicinity of the diagonal. We observe that four bounded derivatives can be taken up to the diagonal. This is joint work with A.V. Sobolev.

Local perturbations perturb locally in weakly interacting quantum spin systems.

Joscha Henheik
IST Austria

Based on a result by Yarotsky (*J. Stat. Phys. 118, 2005*), we prove that localized but otherwise arbitrary perturbations of weakly interacting quantum spin systems with uniformly gapped on-site terms change the ground state of such a system only locally, even if they close the spectral gap. We call this a strong version of the *local perturbations perturb locally (LPPL)* principle which is known to hold for much more general gapped systems, but only for perturbations that do not close the spectral gap of the Hamiltonian. We also extend this strong LPPL-principle to Hamiltonians that have the appropri- ate structure of gapped on-site terms and weak interactions only locally in some region of space. This is joint work with Stefan Teufel and Tom Wessel (*Lett. Math. Phys. 112, 2022*).

Spectral transitions for Dirac operators with δ-shell potentials supported on curves

Markus Holzmann
Graz University of Technology

In this talk the self-adjointness and spectral properties of Dirac operators with singular δ-shell potentials supported on smooth curves in $\mathbb{R}^2$ are discussed. For a three-parameter group of coefficients of the δ-interaction the self-adjoint realizations are described. It turns out that there is a critical combination of coupling constants for which there is a loss of Sobolev regularity in the domain of definition and a spectral transition occurs. More precisely, if the interaction support is a closed and compact curve, then there is an additional point in the essential spectrum in the critical case. If the interactions support is a straight line, then an interval of continuous spectrum collapses to an eigenvalue of infinite multiplicity in the critical case.

Quantum systems at the brink: existence and decay rates of bound states at thresholds

Michal Jex
Université Paris Dauphine-PSL

The stability of quantum systems is directly related to the existence of eigenstates. The situation for eigenstates below the threshold of the essential spectrum is well studied. They exhibit exponential decay, and their existence is linked to the energy gap. However, the situation at the threshold is much more subtle. There are two challenging problems for the states at the threshold-their existence and asymptotic behaviour. Since the usual methods for addressing these problems need a safety distance to the essential spectrum, they cannot be applied in critical cases, when an eigenvalue enters the continuum. We present a new method how to address both problems. As an illustration of the application we derive sharp upper and lower bounds for the asymptotic behaviour of the ground state of critical helium–type systems at the thresh- old of the essential spectrum. This is the first proof of the precise asymptotic behaviour of the ground state for this benchmark problem in quantum chemistry. Moreover, our bounds describe precisely how the asymptotic decay of the ground state changes, when the system becomes critical. In addition, we show the existence of a ground state of this quantum critical system with a finite nuclear mass. Previously this had been known

only in the Born–Oppenheimer approximation of infinite nuclear mass.
Joint with Dirk Hundertmark, Markus Lange.

Asymptotic Floquet theory and its application to perturbation control of Floquet exponents and classification of asymptotic exceptional points

Thea Kosche
ETH Zurich

In this talk a new asymptotic theory for Floquet exponents of time-periodic ordinary differential equations (ODEs) will be presented. This theory gives full asymptotic expansions of the Floquet exponents. In the course of the talk it will serve as the groundwork for the control of Floquet exponent perturbation and as the groundwork for a criterion for the occurrence of asymptotic exceptional points. The classical harmonic oscillator and Floquet metamaterials will be used as an illustration and application of the presented theory. The talk will conclude with a full classification of asymptotic exceptional points in the case of a dimer Floquet metamaterial.

Link to the related paper:
https://www.sciencedirect.com/science/article/pii/S0022039622001450

Phase Transition in the Peierls model for polyacetylene

Adechola Emile Kodjo Kouande
CEREMADE

At null temperature, it is a well-known fact that in closed polyacetylene molecular chains having an even number $(L = 2N)$ of carbon atoms (e.g. benzene), the valence electrons arranged themselves one link in two. This phenomenon is well understood in the Peierls model, introduced in 1930, which is a simple non-linear functional describing polyacetylene chains. In this model, there is a break of symmetry, called Peierls instability: the minimizers are never 1-periodic. Many results regarding this model at null temperature has been obtained, especially in the even case $(L = 2N)$ by Tom Kennedy and Elliott H. Lieb who showed the stability of the Peierls instability and also in the odd case $(L = 2N+1)$ with the works of Mauricio Garcia Arroyo and Eric Séré who showed the existence of a kink state, between the two dimerized configurations. In this talk, we will provide the results obtained by studying this model with temperature. More precisely we will consider the Peierls model for polyactetylene in presence of temperature, we will prove the existence of a critical temperature below which the chain is dimerized or kink- like according to its parity, and above which the system is 1-periodic. The chain behaves like an insulator below the critical temperature and like a metal above it. We will characterize the critical temperature in the thermodynamic limit model and study

the bifurcation around it.
Joint work with David Gontier and Eric Séré.

The BCS energy gap at low and high density

Asbjørn Bækgaard Lauritsen
IST Austria

We study the Bardeen–Cooper–Schrieffer (BCS) energy gap (at zero temperature) Ξ in both the low-and high-density limits. Here we find asymptotic formulas depending on the interaction between the particles. In the low-density limit the dependence is only via the scattering length. We compare these asymptotic formulas with known analogous expressions for the critical temperature T_c and show that in both limits the ratio Ξ/T_c is the same universal constant independent of the interaction.

This talk is based on arXiv:2009.03701 and 2106.02028. This is joint work with Joscha Henheik.

Scattering for Schrödinger operators with potentials concentrated near a subspace

Tal Malinovitch
Yale university

We consider the scattering properties of Schrödinger operators with potentials concentrated near a subspace of $\mathbb{R}^d$. This is one of many models of a quantum particle interacting with a surface. For such operators, we show the existence of scattering states and characterize their orthogonal complement as a set of "surface states
, which consists of states that are confined to the subspace (such as pure point states) and states that escape it at a sublinear rate, in a suitable sense. Our proof uses a novel interpretation of the Enss method [1] in order to obtain a dynamical characterisation of the orthogonal complement of the scattering states. In this talk we will state our results, and sketch some of the main ideas in the proof.
This is a joint work with Adam Black.

[1] Volker Enss. Asymptotic completeness for quantum mechanical potential scattering. Communications in Mathematical Physics, 61(3):285–291, 1978.

Random Schrödinger operators with complex decaying potentials

Konstantin Merz
TU Braunschweig

Estimating the location and accumulation rate of eigenvalues of Schrödinger operators is a classical problem in spectral theory and mathematical physics. The pioneering

work of R. Frank (Bull. Lond. Math. Soc., 2011) illustrated the power of Fourier analytic methods – like the uniform Sobolev inequality by Kenig, Ruiz, and Sogge, or the Stein–Tomas restriction theorem – in this quest, when the potential is non-real and has “short range”. Recently S. Bögli and J.-C. Cuenin (arXiv:2109.06135) showed that Frank’s “short-range” condition is in fact optimal, thereby disproving a conjecture by A. Laptev and O. Safronov (Comm. Math. Phys., 2009) concerning Keller–Lieb–Thirring-type estimates for eigenvalues of Schrödinger operators with complex potentials.
In this talk, we estimate complex eigenvalues of continuum random Schrödinger operators of Anderson type. Our analysis relies on methods of J. Bourgain (Discrete Contin. Dyn. Syst., 2002, Lecture Notes in Math., 2003) related to almost sure scattering for random lattice Schrödinger operators, and allows us to consider potentials which decay almost twice as slowly as in the deterministic case.
The talk is based on joint work with Jean-Claude Cuenin.

Thomas–Fermi profile of a fast rotating Bose–Einstein condensate

Dinh Thi Nguyen
ENS Lyon

We study the minimizers of a magnetic 2D non-linear Schrödinger energy functional in a quadratic trapping potential, describing a rotating Bose–Einstein condensate. We derive an effective Thomas–Fermi-like model in the rapidly rotating limit where the centrifugal force compensates the confinement, and available states are restricted to the lowest Landau level. The coupling constant of the effective Thomas–Fermi functional is to linked the emergence of vortex lattices (the Abrikosov problem). We define it via a low density expansion of the energy of the corresponding homogeneous gas in the thermodynamic limit.

Boundary Superconductivity in the BCS Model

Barbara Roos
IST Austria

We consider the linear BCS equation, determining the BCS critical temperature, in the presence of a boundary, where Dirichlet boundary conditions are imposed. In the one-dimensional case with point interactions, we prove that the critical temperature is strictly larger than the bulk value, at least at weak coupling. In particular, the Cooper-pair wave function localizes near the boundary, an effect that cannot be modeled by effective Neumann boundary conditions on the order parameter as often imposed in Ginzburg-Landau theory. We also show that the relative shift in critical temperature vanishes if the coupling constant either goes to zero or to infinity.

A microscopic derivation of Gibbs measures for the 1D focusing cubic nonlinear Schrödinger equation

Andrew James Rout
University of Warwick

In this work, we give a microscopic derivation of the Gibbs measure and the associated time-dependent correlation functions for the 1D cubic focusing non- linear Schrödinger equation (NLS) from many-body quantum theory. These objects were first studied in the constructive quantum field theory literature in the 1970s and later in the nonlinear dispersive PDE literature by Lebowitz, Rose, and Speer and by Bourgain. Previous derivation results for the corresponding defocusing problems were obtained by Lewin, Nam, and Rougerie and by Fröhlich, Knowles, Schlein, and the second author. This is the first result that treats the focusing regime, i.e. the regime in which the Hamiltonian is not necessarily positive. Due to the focusing nature of the problem, we need to add a suitable truncation in the rescaled number of particles in the quantum problem and on the mass in the classical problem. Our method is based on the perturbative approach developed in the earlier work of Fröhlich, Knowles, Schlein, and the second author. This method has to be adapted to take into account the aforementioned truncation. Our results hold both for the local NLS and for the nonlocal NLS with arbitrary L^p convolution potentials.
This summarises a preprint with Vedran Sohinger.

Point interaction in quantum mechanics revised

Hamidreza Saberbaghi
GSSI

A careful look to the entire family of self-adjoint extensions of the Laplacian known as quantum point interaction Hamiltonians shows that the great majority of them do not become either singular or trivial when the positions of two or more scattering centres tend to coincide. In this sense, those Hamiltonians appear to be "renormalised" by default as opposed to the point interaction Hamiltonians usually considered in the literature and characterised by singular boundary conditions around each scattering centre array. We will summarise some properties of these sub-family of extensions and try to clarify the renormalisation mechanism which makes them regular and physically relevant. We will show how the use of these Hamiltonians in the Born Oppenheimer approximation of the three particle dynamics avoids the unboundedness problem and predicts a correct Efimov spectrum at low energy.
This is a joint work with Rodolfo Figari and Alessandro Teta.

Spectral properties for three-dimensional Schrödinger operators with point interactions

Raffaele Scandone
GSSI

In this talk I will discuss the spectral behavior of Schrödinger operators with finitely-many point interactions in three dimensions. In particular, I will prove the absence of non-zero real resonances and a low-energy expansion for the resolvent, in terms of weighted L^2-spaces. Moreover, I will discuss some consequences for the dispersive and scattering properties of the associated unitary evolution.

Bogoliubov Theory for Trapped Bosons in the Gross-Pitaevskii Regime

Severin Stefan Schraven
The University of British Columbia

We consider systems of N bosons in $\mathbb{R}^3$, trapped by an external potential. The interaction is repulsive and has a scattering length of the order N^{-1} (Gross-Pitaevskii regime). We determine the ground state energy and the low-energy excitation spectrum up to errors that vanish in the limit $N \to \infty$. This is joint work with C. Brennecke and B. Schlein.

Bose-Einstein condensation on hyperbolic spaces

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A well-known open problem in mathematical physics is the question if the Bose gas exhibits condensation in the thermodynamic limit. We show the analog of this conjecture on certain hyperbolic manifolds of growing volume where, in contrast to the Euclidean case, the Laplacian has a uniform gap. Combining this with Dyson's classical upper bound on the energy density transferred to the hyperbolic setting yields a relatively short proof.

Lieb-Robinson bounds and automorphic equivalence for long-range interactions

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In recent years, the automorphic-equivalence of gapped ground states of lattice spin and fermion systems gained attention [e.g. 1, 4, 7, 9]: Let $H^\Lambda(s) = \sum_{Z\subset\Lambda} \Phi(Z, s)$ be a family of short-range Hamiltonians given by exponentially localized interactions $\Phi(\cdot, s)$ on a finite lattice λ with ground states $\rho^\Lambda(s)$ where $s \in [0, 1]$. If the spectrum of $H^\Lambda(s)$ has a spectral gap above the ground state energy uniform in s, then there exists an automorphism $\alpha^\Lambda(s)$ on the algebra of bounded operators satisfying $\rho^\Lambda(s) = \alpha^\Lambda(s)\circ\rho^\Lambda(0)$. The key finding is that for short-range Hamiltonians, the generator of $\alpha^\Lambda(s)$ can be given by an almost exponentially localized interaction. Physically, this means that the ground

state of a gapped system only changes locally after adding a perturbation which does not close the gap. And it in particular allows to take the thermodynamic limit $\Lambda \to \mathbb{Z}^d$ to obtain an automorphism $\alpha(s)$ on the algebra of quasi-local operators providing an equivalence between the ground states of the infinite systems. Moreover, it allows to prove that local perturbations perturb locally [1]: As long as $\partial/\partial s H^\Lambda(s)$ is supported in a certain region, the ground state will only change around this region. The main technical ingredients for these locality results are Lieb-Robinson bounds [5, 8, 9].
More recently, there was progress [2, 3, 6] on Lieb-Robinson bounds for long-range Hamiltonians given by polynomially-decaying interactions. We improve these bounds and show that the results on automorphic equivalence of gapped ground states also hold in systems with polynomially-decaying interactions. In particular, the automorphism is generated by a polynomially-decaying interaction (with smaller exponent) and one can again take the thermodynamic limit and lift it to an automorphism on the algebra of quasi-local operators.

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